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The hydrogen atom revisited

Randell L. Mills*

BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ 08512, USA

Abstract

Several myths about quantum mechanics exist due to a loss of awareness of its details since its inception in the beginning of the last century or based on recent experimental evidence. It is taught in textbooks that atomic hydrogen cannot go below the ground state. Atomic hydrogen having an experimental ground state of 13.6 eV can only exist in a vacuum or in isolation, and atomic hydrogen cannot go below this ground state in isolation. However, there is no known composition of matter containing hydrogen in the ground state of 13.6 eV. It is a myth that hydrogen has a theoretical ground state based on first principles. Historically, there were many directions in which to proceed to solve a wave equation for hydrogen. The Schrodinger equation gives the observed spontaneously radiative states and the nonradiative energy level of atomic hydrogen. On this basis alone, it is justified despite its inconsistency with physical laws as well as with many experiments. A solution compatible with first principles and having first principles as the basis of quantization was never found. Scattering results required the solution to be interpreted as probability waves that give rise to the uncertainty principle which in turn forms the basis of the wave particle duality. The correspondence principle predicts that quantum predictions must approach classical predictions on a large scale. However, recent data has shown that the Heisenberg uncertainty principle as the basis of the wave particle duality and the correspondence principle taught in textbooks are experimentally incorrect. Recently, a reconsideration of the postulates of quantum mechanics, has given rise to a closed form solution of a Schrodinger-like wave equation based on first principles. Hydrogen at predicted lower energy levels has been identified in the extreme ultraviolet emission spectrum from interstellar medium. In addition, new compositions of matter containing hydrogen at predicted lower energy levels have recently been observed in the laboratory, which energy levels are achieved using the novel catalysts. © 2000 International Association for Hydrogen Energy. Published by Elsevier Science Ltd. All rights reserved.

1. Introduction

J.J. Balmer showed, in 1885, that the frequencies for some of the lines observed in the emission spectrum of atomic hydrogen could be expressed with a completely empirical relationship. This approach was later extended by J.R. Rydberg, who showed that all of the spectral lines of atomic hydrogen were given by the equation

$$\bar{\nu} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \quad (1)$$

where $R = 109,677 \text{ cm}^{-1}$, $n_f = 1, 2, 3, \dots$, $n_i = 2, 3, 4, \dots$, and $n_i > n_f$. Niels Bohr, in 1913, developed a theory for atomic hydrogen based on an unprecedented postulate of

stable circular orbits that do not radiate. Although no explanation was offered for the existence of stability for these orbits, the results gave energy levels in agreement with Rydberg's equation

$$E_n = -\frac{e^2}{n^2 8\pi\epsilon_0 a_H} = -\frac{13.598 \text{ eV}}{n^2}, \quad (2)$$

$$n = 1, 2, 3, \dots, \quad (3)$$

where a_H is the Bohr radius for the hydrogen atom (52.947 pm), e is the magnitude of the charge of the electron, and ϵ_0 is the vacuum permittivity. Bohr's theory was a straightforward application of Newton's laws of motion and Coulomb's law of electric force — both pillars of classical physics and is in accord with the experimental observation that atoms are stable. However, it is not in accord with electromagnetic theory — another pillar of classical physics which predicts that accelerated charges radiate energy in the form

* Tel.: 00-1-609-490-1090; fax: 00-1-609-490-1066.

E-mail address: rmills@blacklightpower.com (R.L. Mills).

of electromagnetic waves. An electron pursuing a curved path is accelerated and therefore should continuously lose energy, spiraling into the nucleus in a fraction of a second. The predictions of electromagnetic theory have always agreed with experiment, yet atoms do not collapse. To the early 20th century theoreticians, this contradiction could mean only one thing: The laws of physics that are valid in the macroworld do not hold true in the microworld of the atom. In 1923, de Broglie suggested that the motion of an electron has a wave aspect — $\lambda = h/p$. This concept seemed unlikely according to the familiar properties of electrons such as charge, mass and adherence to the laws of particle mechanics. But, the wave nature of the electron was confirmed by Davisson and Germer in 1927 by observing diffraction effects when electrons were reflected from metals. Schrödinger reasoned that if electrons have wave properties, there must be a wave equation that governs their motion. And in 1926, he proposed the Schrödinger equation, $H\Psi = E\Psi$, where Ψ is the wave function, H is the wave operator, and E is the energy of the wave. This equation, and its associated postulates, is now the basis of *quantum mechanics*, and it is the basis for the world view that the atomic realm including the electron and photon cannot be described in terms of “pure” wave and “pure” particle but in terms of a wave-particle duality. The wave-particle duality based on the fundamental principle that physics on an atomic scale is very different from physics on a macroscopic scale is central to present day atomic theory [1].

2. Development of atomic theory

2.1. Bohr theory

In 1911, Rutherford proposed a planetary model for the atom where the electrons revolved about the nucleus (which contained the protons) in various orbits to explain the spectral lines of atomic hydrogen. There was, however, a fundamental conflict with this model and the prevailing classical physics. According to classical electromagnetic theory, an accelerated particle radiates energy (as electromagnetic waves). Thus, an electron in a Rutherford orbit, circulating at constant speed but with a continually changing direction of its velocity vector is being accelerated; thus, the electron should constantly lose energy by radiating and spiral into the nucleus.

An explanation was provided by Bohr in 1913, when he assumed that the energy levels were quantized and the electron was constrained to move in only one of a number of allowed states. Niels Bohr's theory for atomic hydrogen was based on an unprecedented postulate of stable circular orbits that do not radiate. Although no explanation was offered for the existence of stability for these orbits, the results gave energy levels in agreement with Rydberg's equation. Bohr's theory was a straightforward application of Newton's laws

of motion and Coulomb's law of electric force. According to Bohr's model, the point particle electron was held to a circular orbit about the relatively massive point particle nucleus by the balance between the coulombic force of attraction between the proton and the electron and centrifugal force of the electron.

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{m_e v^2}{r} \quad (4)$$

Bohr postulated the existence of stable orbits in defiance of classical physics (Maxwell's equations), but he applied classical physics according to Eq. (4). Then Bohr realized that the energy formula (Eqs. (2) and (3)) was given by postulating nonradiative states with angular momentum

$$L_z = m_e v r = n\hbar, \quad n = 1, 2, 3, \dots \quad (5)$$

and by solving the energy equation classically. The Bohr radius is given by substituting the solution of Eq. (5) for v into Eq. (4).

$$r = \frac{4\pi\epsilon_0 \hbar^2 n^2}{m_e e^2} = n^2 a_0, \quad n = 1, 2, 3, \dots \quad (6)$$

The total energy is the sum of the potential energy and the kinetic energy. In the present case of an inverse squared central field, the total energy (which is the negative of the binding energy) is one-half the potential energy [2]. The potential energy, $\phi(r)$, is given by Poisson's equation

$$\phi(r) = - \int_{V'} \frac{\rho(r') dv'}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|} \quad (7)$$

For a point charge at a distance r from the nucleus the potential is

$$\phi(r) = - \frac{e^2}{4\pi\epsilon_0 r} \quad (8)$$

Thus, the total energy is given by

$$E = - \frac{Z^2 e^2}{8\pi\epsilon_0 r}, \quad (9)$$

where $Z = 1$. Substitution of Eq. (6) into Eq. (9) with the replacement of the electron mass by the reduced electron mass gives Eqs. (2) and (3).

Bohr's model was in agreement with the observed hydrogen spectrum, but it failed with the helium spectrum, and it could not account for chemical bonds in molecules. The prevailing wisdom was that the Bohr model failed because it was based on the application of Newtonian mechanics for discrete particles. Its limited applicability was attributed to the unwarranted assumption that the energy levels are quantized.

Bohr's theory may also be analyzed according to the corresponding energy equation. Newton's differential equations of motion in the case of the central field such as a gravitational or electrostatic field are

$$m(\ddot{r} - r\dot{\theta}^2) = f(r), \quad (10)$$

$$m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = 0, \quad (11)$$

where $f(r)$ is the central force. The second or transverse equation, Eq. (11), gives the result that the angular momentum is constant.

$$r^2\dot{\theta} = \text{constant} = L/m, \quad (12)$$

where L is the angular momentum. The central force equations can be transformed into an orbital equation by the substitution, $u = 1/r$. The differential equation of the orbit of a particle moving under a central force is

$$\frac{\delta^2 u}{\delta \theta^2} + u = \frac{-1}{mL^2 u^2 / m^2} f(u^{-1}). \quad (13)$$

Because the angular momentum is constant, motion in only one plane need be considered; thus, the orbital equation is given in polar coordinates. The solution of Eq. (13) for an inverse square force

$$f(r) = -\frac{k}{r^2} \quad (14)$$

is

$$r = r_0 \frac{1 + e}{1 + e \cos \theta}, \quad (15)$$

$$e = A \frac{mL^2 / m^2}{k}, \quad (16)$$

$$r_0 = \frac{mL^2 / m^2}{k(1 + e)}, \quad (17)$$

where e is the eccentricity and A is a constant. The equation of motion due to a central force can also be expressed in terms of the energies of the orbit. The square of the speed in polar coordinates is

$$v^2 = (\dot{r}^2 + r^2\dot{\theta}^2). \quad (18)$$

Since a central force is conservative, the total energy, E , is equal to the sum of the kinetic, T , and the potential, V , and is constant. The total energy is

$$\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + V(r) = E = \text{constant}. \quad (19)$$

Substitution of the variable $u = 1/r$ and Eq. (12) into Eq. (19) gives the orbital energy equation

$$\frac{1}{2}m\frac{L^2}{m^2} \left[\left(\frac{\delta^2 u}{\delta \theta^2} \right) + u^2 \right] + V(u^{-1}) = E. \quad (20)$$

Because the potential energy function $V(r)$ for an inverse square force field is

$$V(r) = -\frac{k}{r} = -ku, \quad (21)$$

the energy equation of the orbit (Eq. (20)), is

$$\frac{1}{2}m\frac{L^2}{m^2} \left[\left(\frac{\delta^2 u}{\delta \theta^2} \right) + u^2 \right] - ku = E, \quad (22)$$

$$\left[\frac{\delta^2 u}{\delta \theta^2} + u^2 \right] - \frac{2m}{L^2} [E + ku] = 0, \quad (23)$$

which has the solution

$$r = \frac{m(L^2/m^2)k^{-1}}{1 + [1 + 2Em(L^2/m^2)k^{-2}]^{1/2} \cos \theta}, \quad (24)$$

where the eccentricity, e is

$$e = \left[1 + 2Em\frac{L^2}{m^2}k^{-2} \right]^{1/2}. \quad (25)$$

Eq. (25) permits the classification of the orbits according to the total energy, E , as follows:

$E < 0$, $e < 1$, ellipse,

$E < 0$, $e = 0$, circle (special case of ellipse),

$E = 0$, $e = 1$, parabolic orbit,

$E > 0$, $e > 1$, hyperbolic orbit. (26)

Since $E = T + V$ and is constant, the closed orbits are those for which $T < |V|$, and the open orbits are those for which $T \geq |V|$. It can be shown that the time average of the kinetic energy, $\langle T \rangle$, for elliptic motion in an inverse square field is $1/2$ that of the time average of the potential energy, $\langle V \rangle$. $\langle T \rangle = 1/2 \langle V \rangle$.

Bohr's solution is trivial in that he specified a circular bound orbit which determined that the eccentricity was zero, and he specified the angular momentum as a integer multiple of Planck's constant bar. Eq. (25) in CGS units becomes

$$E = -\frac{1}{2} \frac{me^4}{n^2 \hbar^2} = -\frac{e^2}{2n^2 a_0}. \quad (27)$$

2.2. Schrödinger theory of the hydrogen atom

In 1923, de Broglie suggested that the motion of an electron has a wave aspect — $\lambda = h/p$. This was confirmed by Davisson and Germer in 1927 by observing diffraction effects when electrons were reflected from metals. Schrödinger reasoned that if electrons have wave properties, there must be a wave equation that governs their motion. And, in 1926, he proposed the time-independent Schrödinger equation

$$H\Psi = E\Psi, \quad (28)$$

where Ψ is the wave function, H is the wave operator, and E is the energy of the wave. To give the sought three quantum numbers, the Schrödinger equation solutions are three dimensional in space and four dimensional in spacetime

$$\left[\nabla^2 - \frac{1}{v^2} \frac{\delta^2}{\delta t^2} \right] \Psi(r, \theta, \phi, t) = 0, \quad (29)$$

where $\Psi(r, \theta, \phi, t)$ according to quantum theory is the probability density function of the electron as described below. When the time harmonic function is eliminated [3]

$$\begin{aligned} -\frac{\hbar^2}{2\mu} \left[\frac{1}{r^2} \frac{\delta}{\delta r} \left(r^2 \frac{\delta \Psi}{\delta r} \right) + \frac{1}{r^2 \sin \theta} \frac{\delta}{\delta \theta} \left(\sin \theta \frac{\delta \Psi}{\delta \theta} \right) \right]_{r,\theta} \\ + \left[\frac{\delta^2 \Psi}{\delta \phi^2} \right]_{r,\theta} + V(r) \Psi(r, \theta, \phi) = E \Psi(r, \theta, \phi), \end{aligned} \quad (30)$$

where the potential energy $V(r)$ in CGS units is

$$V(r) = -\frac{e^2}{r}. \quad (31)$$

The Schrödinger equation (Eq. (30)) can be transformed into a sum comprising a part that depends only on the radius and a part that is a function of angle only obtained by separation of variables and linear superposition in spherical coordinates. The general form of the solutions for $\psi(r, \theta, \phi)$ are

$$\psi(r, \theta, \phi) = \sum_{l,m} R_{nlm}(r) Y_{lm}(\theta, \phi), \quad (32)$$

where l and m are separation constants. The azimuthal (θ) part of Eq. (30) is the generalized Legendre equation which is derived from the Laplace equation by Jackson (Eq. (3.9) of Jackson [4]). The solutions for the full angular part of Eq. (30), $Y_{lm}(\theta, \phi)$, are the spherical harmonics

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos \theta) e^{im\phi}. \quad (33)$$

By substitution of the eigenvalues corresponding to the angular part [5], the Schrödinger equation becomes the radial equation, $R(r)$, given by

$$-\frac{\hbar^2}{2mr^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left[\frac{\hbar^2 l(l+1)}{2mr^2} + V(r) \right] R(r) = ER(r). \quad (34)$$

The time-independent Schrödinger equation is similar to Eq. (20) except that the solution is for the distribution of a spatial wavefunction in three dimensions rather than the dynamical motion of a point particle of mass m along a one-dimensional trajectory. Electron motion is implicit in the Schrödinger equation. For wave propagation in three dimensions, the full time-dependent Schrödinger equation is required, whereas the classical case contains time

derivatives. The kinetic energy of rotation is K_{rot} is given classically by

$$K_{\text{rot}} = \frac{1}{2} m r^2 \omega^2, \quad (35)$$

where m is the mass of the electron. In the time-independent Schrödinger equation, the kinetic energy of rotation K_{rot} is given by

$$K_{\text{rot}} = \frac{\ell(\ell+1)\hbar^2}{2mr^2}, \quad (36)$$

where

$$L = \sqrt{\ell(\ell+1)\hbar^2} \quad (37)$$

is the magnitude of the electron angular momentum L for the state $Y_{lm}(\theta, \phi)$.

In the case of the ground state of hydrogen, the Schrödinger equation solution is trivial for an implicit circular bound orbit which determines that the eccentricity is zero, and with the specification that the electron angular momentum is Planck's constant bar. With $k = e^2$, Eq. (25) in CGS units becomes

$$E = -\frac{1}{2} \frac{m e^4}{\hbar^2} = -\frac{e^2}{2a_0}, \quad (38)$$

which corresponds to $n = 1$ in Eq. (27). Many problems in classical physics give three quantum numbers when three spatial dimensions are considered. In order to obtain three quantum numbers, the Schrödinger equation requires that the solution is for the distribution of a spatial wavefunction in three dimensions with implicit motion rather than a one dimensional trajectory of a point particle as shown below. However, this approach gives rise to predictions about the angular momentum and angular energy which are not consistent with experimental observations as well as a host of other problems which are summarized in Section 10.

The radial equation may be written as

$$\frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \frac{2mr^2}{\hbar^2} \left[E - V(r) - \frac{l(l+1)\hbar^2}{2mr^2} \right] R(r) = 0. \quad (39)$$

Let $U(r) = rR(r)$, then the radial equation reduces to

$$U'' + \frac{2m}{\hbar^2} \left[E - V(r) - \frac{l(l+1)\hbar^2}{2mr^2} \right] U = 0, \quad (40)$$

where

$$\psi = \frac{1}{r} U_{lm}(r) Y_{lm}(\theta, \phi). \quad (41)$$

Substitution of the potential energy given by Eq. (31) into Eq. (40) gives for sufficiently large r

$$U'' - \left(\frac{\alpha}{2} \right)^2 U = 0 \quad (42)$$

provided we define

$$\left(\frac{\alpha}{2} \right)^2 = \frac{-2mE}{\hbar^2}, \quad (43)$$

where α is the eigenvalue of the eigenfunction solution of the Schrödinger equation given *infra* having units of reciprocal length and E is the energy levels of the hydrogen atom. To arrive at the solution which represents the electron, a suitable boundary condition must be imposed. Schrödinger postulated a boundary condition: $\Psi \rightarrow 0$ as $r \rightarrow \infty$, which leads to a purely mathematical model of the electron. This equation is not based on first principles, has no validity as such, and should not be represented as so. The right-hand side of Eq. (43) must be *postulated* in order that the Rydberg equation is obtained as shown below. The postulate is implicit since Eq. (43) arises from the Schrödinger equation which is postulated. It could be defined *arbitrarily*, but is justified because it gives the Rydberg formula. That Schrödinger guessed the accepted approach is not surprising since many approaches were contemplated at this time [6], and since none of these approaches were superior, Schrödinger's approach prevailed.

The solution of Eq. (42) that is consistent with the boundary condition is

$$U_\infty = c_1 e^{(\alpha/2)r} + c_2 e^{-(\alpha/2)r} \tag{44}$$

In the case that α is real, the energy of the particle is negative. In this case U_∞ will not have an integrable square if c_1 fails to vanish wherein the radial integral has the form

$$\int_0^\infty R^2 r^2 dr = \int U_\infty^2 dr \tag{45}$$

It is shown below that the solution of the Schrödinger corresponds to the case wherein c_1 fails to vanish. Thus, the solutions with sufficiently large r are infinite. The same problem arises in the case of a free electron that is ionized from hydrogen. If α is imaginary, which means that E is positive, Eq. (42) is the equation of a linear harmonic oscillator [7]. U_∞ shows sinusoidal behavior; thus, the wavefunction for the free electron cannot be normalized and is infinite. In addition, the angular momentum of the free electron is infinite since it is given by $\ell(\ell + 1)\hbar^2$ (Eq. (37)) where $\ell \rightarrow \infty$.

In order to solve the bound electron states, let

$$E = -W \tag{46}$$

so that W is positive. In Eq. (39), let $r = x/\alpha$ where α is given by Eq. (43)

$$x \frac{d^2 R}{dx^2} + 2 \frac{dR}{dx} + \left[\frac{2me^2}{\hbar^2 \alpha} - \frac{x}{4} - \frac{l(l+1)}{x} \right] R = 0 \tag{47}$$

Eq. (47) is the differential equation for associated Laguerre functions given in general form by

$$xy'' + 2y' + \left[n^* - \frac{k-1}{2} - \frac{x}{4} - \frac{k^2-1}{4x} \right] y = 0, \tag{48}$$

which has a solution possessing an integrable square of the form

$$y = e^{-x/2} x^{(k-1)/2} \frac{d^k}{dx^k} L_{n^*}(x), \tag{49}$$

provided that n^* and k are positive integers. However, n^* does not have to be an integer, it may be any *arbitrary* constant β . Then the corresponding solution is [8]

$$y = e^{-x/2} x^{(k-1)/2} \frac{d^k}{dx^k} L_\beta(x). \tag{50}$$

In the case that n^* is chosen to be an integer in order to obtain the Rydberg formula, $n^* - k \geq 0$ since otherwise $L_{n^*}^k(x)$ of Eq. (49) would vanish. By comparing Eqs. (47) and (48),

$$\frac{k^2 - 1}{4} = \ell(\ell + 1). \tag{51}$$

Thus,

$$k = 2\ell + 1 \tag{52}$$

and

$$n^* - \frac{k-1}{2} = n^* - \ell = \frac{me^2}{\hbar} \left(\frac{\alpha}{2} \right)^{-1} \tag{53}$$

Substitution of the value of α and solving for W gives

$$W = \frac{1}{2} \frac{me^4}{(n^* - \ell)^2 \hbar^2} \tag{54}$$

Because of the conditions on n^* and k , the quantity $n^* - \ell$ cannot be zero. It is usually denoted by n and called the principal quantum number. The energy states of the hydrogen atom are

$$W_n = -E_n = \frac{1}{2} \frac{me^4}{n^2 \hbar^2} \tag{55}$$

and the corresponding eigenfunctions from Eq. (49) are

$$R_{n,\ell} = c_{n,\ell} e^{-x/2} x^\ell L_{n-\ell}^{2\ell+1}(x), \tag{56}$$

where the variable x is defined by

$$x = \alpha r = \frac{\sqrt{8mW}}{\hbar} r = \frac{2me^2}{n\hbar^2} r. \tag{57}$$

In the Bohr theory of the hydrogen atom, the first orbital has a radius in CGS units given by

$$a_0 = \frac{\hbar^2}{me^2} = 0.53 \times 10^{-8} \text{ cm.} \tag{58}$$

Thus, $\alpha = 2/na_0$ and

$$x = \frac{2}{n} \frac{r}{a_0} \tag{59}$$

The energy states of the hydrogen atom in CGS units in terms of the Bohr radius are given by Eq. (27). From

Eq. (56), $R_{n,\ell}$ for the hydrogen atom ground state is

$$R_{1,0} = c_{1,0} e^{-r/a_0} L_1^1 = 2a_0^{-3/2} e^{-r/a_0}. \quad (60)$$

For this state

$$Y_{00} = \text{const} \tan t = (4\pi)^{-1/2}, \quad (61)$$

when the function is normalized. Thus, the ground state function is

$$\psi_0 = (\pi a_0^3)^{-1/2} e^{-r/a_0}. \quad (62)$$

Immediately further problems arise. Since ℓ must equal zero in the ground state, the predicted angular energy and angular momentum given by Eqs. (36) and (37), respectively, are zero which are experimentally incorrect. In addition, different integer values of ℓ exist in the case of excited electron states. In these cases, the Schrödinger equation solutions (Eqs. (36) and (37)), predict that the excited state rotational energy levels are nondegenerate as a function of the ℓ quantum number even in the absence of an applied magnetic field. Consider the case of the excited state with $n = 2$; $\ell = 1$ compared to the experimentally degenerate state $n = 2$; $\ell = 0$. According to Eq. (37) the difference in angular energy of these two states is 3.4 eV where the expectation radius, $4a_0$, is given by the squared integral of Eq. (70) over space. Thus, the predicted rotational energy in the absence of a magnetic field is over six orders of magnitude of the observed nondegenerate energy (10^{-7} – 10^{-6} eV) in the presence of a magnetic field.

Schrödinger realized that his equation was limited. It is not Lorentzian invariant; thus, it violates special relativity. It also does not comply with Maxwell's equations and other first principle laws. Schrödinger sought a resolution of the incompatibility with special relativity for the rest of his life. He was deeply troubled by the physical consequences of his equation and its solutions. His hope was that the resolution would make his equation fully compatible with classical physics and the quantization would arise from first principles.

Quantum mechanics failed to predict the results of the Stern–Gerlach experiment which indicated the need for an additional quantum number. Quantum electrodynamics was proposed by Dirac in 1926 to provide a generalization of quantum mechanics for high energies in conformity with the theory of special relativity and to provide a consistent treatment of the interaction of matter with radiation. From Weisskopf [9], "Dirac's quantum electrodynamics gave a more consistent derivation of the results of the correspondence principle, but it also brought about a number of new and serious difficulties". Quantum electrodynamics: (1) does not explain nonradiation of bound electrons; (2) contains an internal inconsistency with special relativity regarding the classical electron radius — the electron mass corresponding to its electric energy is infinite; (3) it admits solutions of negative rest mass and negative kinetic energy; (4) the interaction of the electron with the predicted zero-point field

fluctuations leads to infinite kinetic energy and infinite electron mass; and (5) Dirac used the unacceptable states of negative mass for the description of the vacuum; yet, infinities still arise.

A physical interpretation of Eq. (28) was sought. Schrödinger interpreted $e\Psi^*(x)\Psi(x)$ as the charge-density or the amount of charge between x and $x + dx$ (Ψ^* is the complex conjugate of Ψ). Presumably, then, he pictured the electron to be spread over large regions of space. Three years after Schrödinger's interpretation, Max Born, who was working with scattering theory, found that this interpretation led to logical difficulties, and he replaced the Schrödinger interpretation with the probability of finding the electron between r, θ, ϕ and $r + dr, \theta + d\theta, \phi + d\phi$ as

$$\int \Psi(r, \theta, \phi) \Psi^*(r, \theta, \phi) dr d\theta d\phi. \quad (63)$$

Born's interpretation is generally accepted. Nonetheless, interpretation of the wave function is a never-ending source of confusion and conflict. Many scientists have solved this problem by conveniently adopting the Schrödinger interpretation for some problems and the Born interpretation for others. This duality allows the electron to be everywhere at one time — yet have no volume. Alternatively, the electron can be viewed as a discrete particle that moves here and there (from $r = 0$ to ∞), and $\Psi\Psi^*$ gives the time average of this motion.

Schrödinger was also troubled by the philosophical consequences of his theory since quantum mechanics leads to certain philosophical interpretations [10] which are not sensible. Some conjure up multitudes of universes including "mind" universes; others require belief in a logic that allows two contradictory statements to be true. The question addressed is whether the universe is determined or influenced by the possibility of our being conscious of it. The meaning of quantum mechanics is debated, but the Copenhagen interpretation is predominant. It asserts that "what we observe is all we can know; any speculation about what a photon, an atom, or even a superconducting quantum interference device (SQUID) really is or what it is doing when we are not looking is just that — speculation" [10]. As shown by Platt [11] in the case of the Stern–Gerlach experiment, "the postulate of quantum measurement (which) asserts that the process of measuring an observable forces the state vector of the system into an eigenvector of that observable, and the value measured will be the eigenvalue of that eigenvector". According to this interpretation every observable exists in a state of superposition of possible states, and observation or the potential for knowledge causes the wavefunction corresponding to the possibilities to collapse into a definite.

According to the quantum mechanical view, a moving particle is regarded as a wave group. To regard a moving particle as a wave group implies that there are fundamental limits to the accuracy with which such "particle" properties as position and momentum can be measured. Quantum predicts that the particle may be located anywhere within its