Fractional Energy States of Hydrogen

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Goals

- Derivation of Fractional Principal Quantum States for solutions of:
  - Non-relativistic Schrödinger equation
  - Relativistic spin-zero version Klein-Gordon equation
  - Implications of n, the principal quantum number, having the dual behavior of:

    n = 1, 2, 3, ... 137

    n = 1/p, where p = 2, 3, 4, 5, ... 137
- Introduce Meta-Energy p-ladder, bridging Chemical to Nuclear Energy gap as a possible model for LENR reactions

Historical Background

- In 1956 Schroedinger wrote in a letter to Wolfgang Yourgrau*

"Admittedly the Schroedinger theory, relativistically framed (without spin) gives a formal expression for the fine structure formula of Sommerfeld, but it is incorrect owing to the appearance of half-integbers instead of integers. My paper in which this is shown has ... never been published. It was withdrawn by me and replaced by the non-relativistic treatment."

*Yourgrau,W and Mandelstram,S (1968) Variational Principles in Dynamics and Quantum Theory, 3rd Ed London, Issac Pitman and Sons

W.J. Moore, Schrödinger: Life and Thought, 1st edition, Cambridge University Press, pp.196-197, 1992

Elongated Narrative can be found at septorcorp.com

Fractional Solutions to Schrodinger Equation

$$x\frac{d^2R}{dx^2} + 2\frac{dR}{dx} + \left[\frac{2me^2}{\hbar^2\alpha} - \frac{x}{4} - \frac{l\,(l+1)}{x}\right] R \;=\; 0$$

L. Schiff, Quantum Mechanics, International Series in Pure and Applied Math., McGraw-Hill, New York, (1955) pp 80-85 Equations 16.6 -16.8.

$$x = \alpha r = \rho \qquad \propto^2 = \frac{8\mu|E|}{\hbar^2}$$

μ = reduced mass
E = eigenvalue

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Fractional Solutions to Schrodinger Equation

$$x\frac{d^2R}{dx^2} + 2\frac{dR}{dx} + \left[\frac{2me^2}{\hbar^2\alpha} - \frac{x}{4} - \frac{l\,(l+1)}{x}\right]R \;=\; 0$$

Substituting $n^* - \frac{k-1}{2} = \frac{2me^2}{\hbar^2\alpha}$ $\quad \frac{k^2-1}{4} = l(l+1)$

We get $$xy'' + 2y' + \left[n^* - \frac{k-1}{2} - \frac{x}{4} - \frac{k^2-1}{4x}\right]y = 0$$

Eqn. 2.71 Henry Margenau and George M. Murphy *The Mathematics of Physics and Chemistry, New York , d. Van Nostrand Company Inc p.78 1943 . Physics Department and Chemistry Department, Yale University.*

Fractional Solutions to Schrodinger Equation

$$xy'' + 2y' + \left[n^* - \frac{k-1}{2} - \frac{x}{4} - \frac{k^2-1}{4x}\right] y = 0$$

Where solution y depends on the value on n^*:

If n^* is a positive integer, y is an associated Laguerre function

$$y = e^{-x/2}\, x^{(k-1)/2}\, L_{n^*}^{k}(x)$$

$$L_{n^*}^{k}(x) \equiv \frac{d^k}{dx^k} L_{n^*}(x)$$

If n^* is any constant, y becomes

$$y = e^{-x/2}\, x^{(k-1)/2}\, \frac{d^k}{dx^k} L_{n^*}(x)$$

Fractional State Derivation Highlights

$$xy'' + 2y' + \left[n^* - \frac{k-1}{2} - \frac{x}{4} - \frac{k^2-1}{4x}\right] y = 0$$

Set:

$$\frac{k^2-1}{4} = l(l+1)$$

$$k^2 - 1 = 4l(l+1) = 4l^2 + 4l + 1 - (2l+1)^2$$

$$k = 2l + 1$$

Fractional State Derivation Highlights

$$E = -W = -\frac{me^4}{2\hbar^2}\frac{1}{(n^* - l)^2}$$

$$n^* = \frac{1}{\mathrm{p}} + l$$

p = 2, 3, 4 …

$$a_0 = \frac{\hbar^2}{me^2}$$

$$E = -\frac{1}{2}\left(\frac{e^2}{a_0}\right)\left(\frac{1}{(\frac{1}{\mathrm{p}})^2}\right)$$

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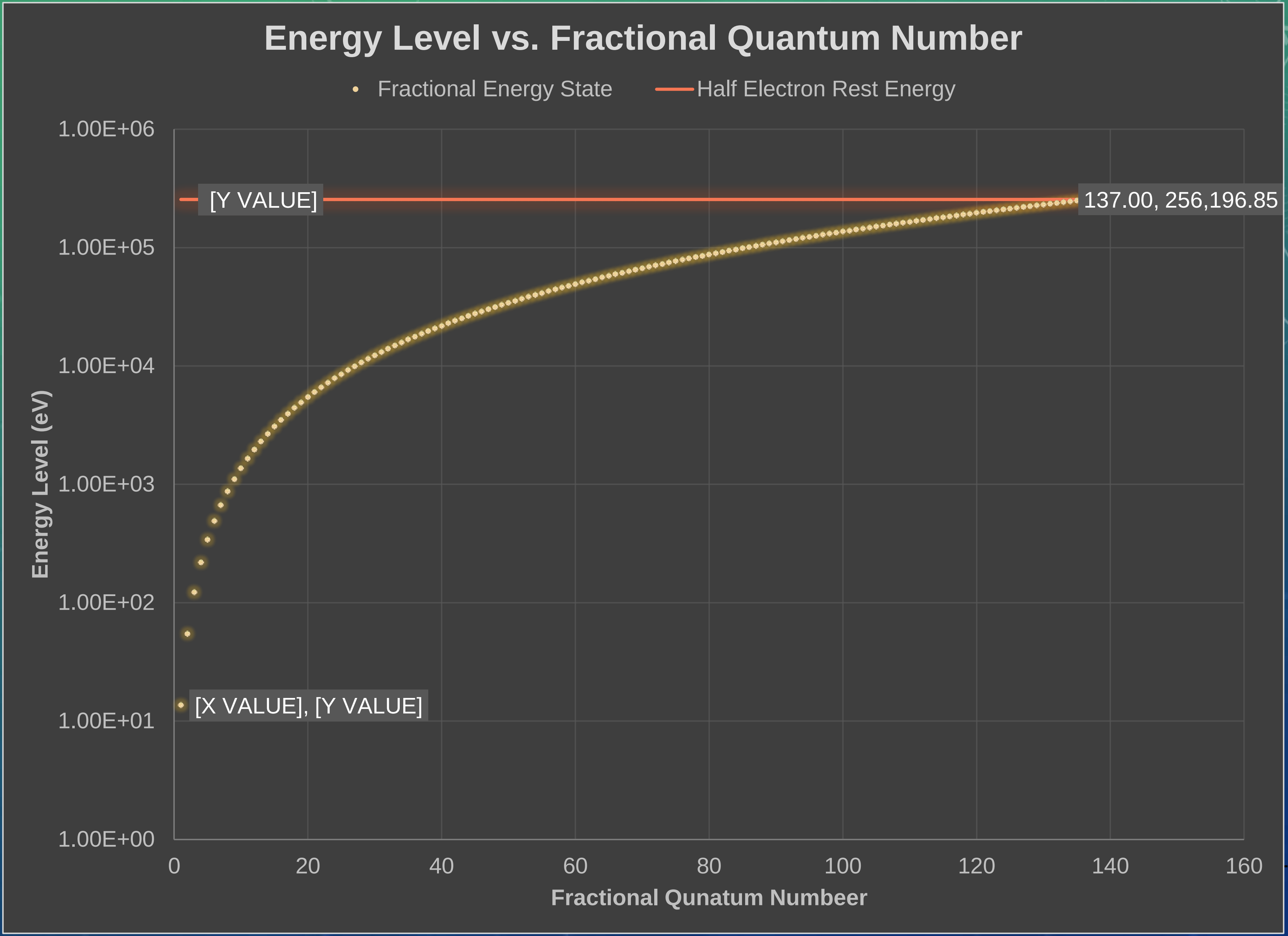
Integer Solutions to Schrodinger Equation

$$E = -W = -\frac{me^4}{2\hbar^2}\frac{1}{(n^* - l)^2}$$

$$n^* = n + l, \quad n = 1, 2, 3 \ldots$$

$$a_0 = \frac{\hbar^2}{me^2}$$

$$E = -\frac{1}{2}\left(\frac{e^2}{a_0}\right)\left(\frac{1}{n^2}\right)$$


Energy Level vs. Fractional Quantum Number
Fractional Energy State
Half Electron Rest Energy
[Y VALUE]
137.00, 256,196.85
[X VALUE], [Y VALUE]
1.00E+06
1.00E+05
1.00E+04
1.00E+03
1.00E+02
1.00E+01
1.00E+00
Energy Level (eV)
0
20
40
60
80
100
120
140
160
Fractional Qunatum Numbeer

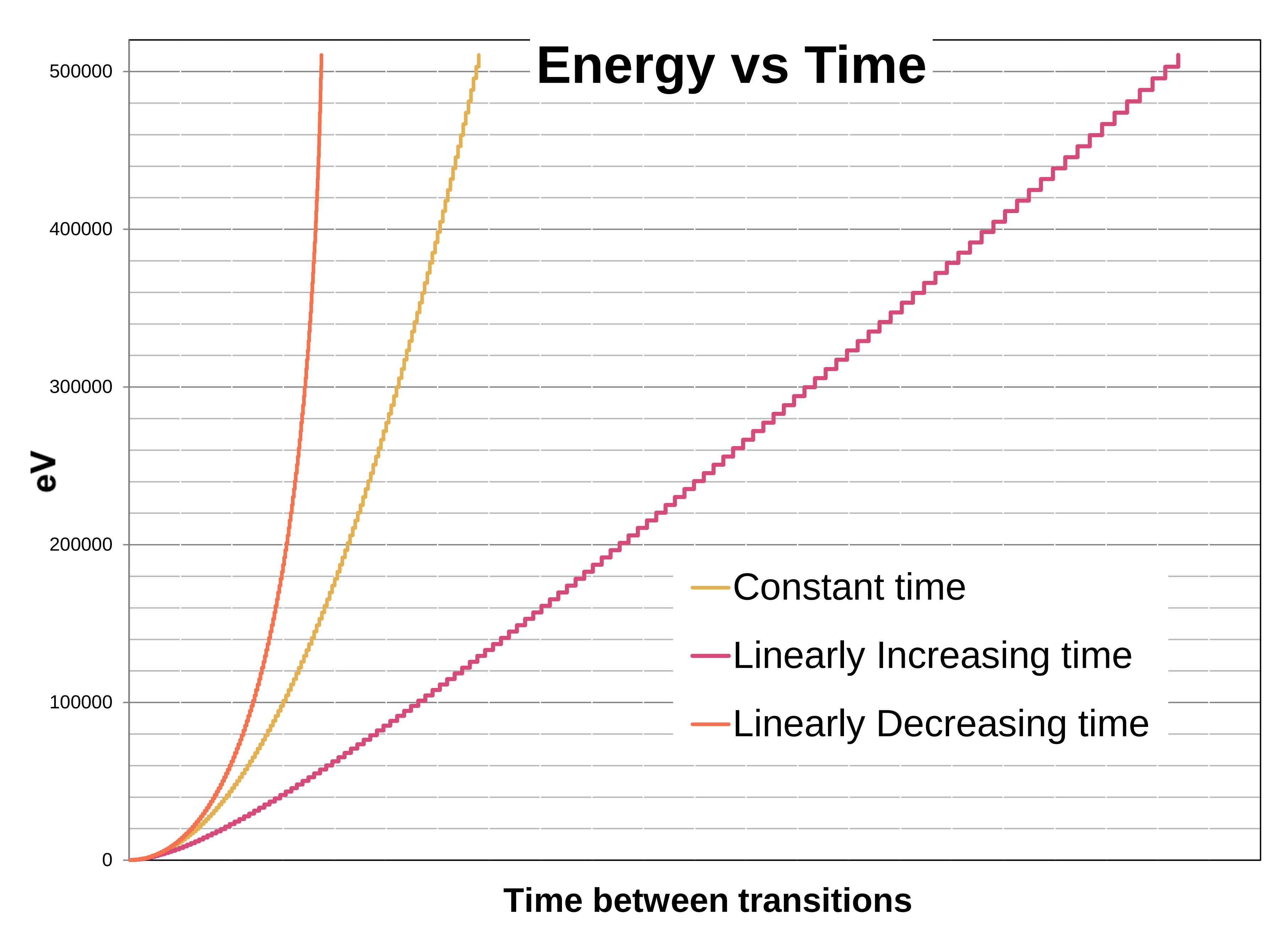
Radii of Fractional Quantum States

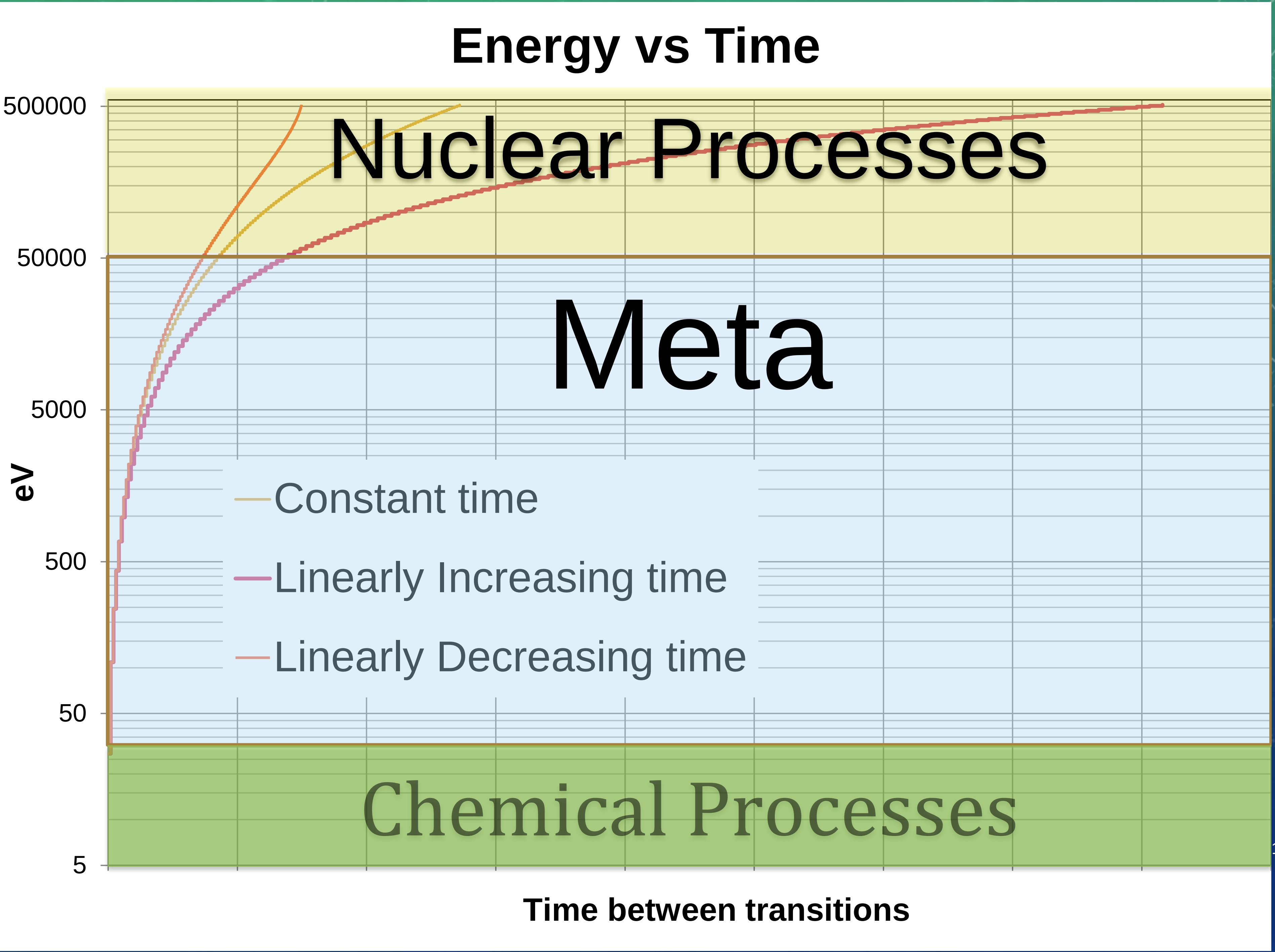
$r_p = \mathbf{a_o}/p$, p = 2, 3, 4..., 137 [$r_n = \mathbf{a_o n}$, n = 1, 2, 3, 4..., 137]**

$\frac{a_o}{137}$ ... $\frac{a_o}{3}$ $\frac{a_o}{2}$ a_o $137a_o$

386 fm **52.9 pm** **7.25 nm**

****T.L. Gill, T. Morris, S.K. Kurtz, Foundations for proper-time relativistic quantum theory, J. Phys. Conf. Ser. 615, 012013 (2015)**


Energy vs Time
500000
400000
300000
200000
100000
0
eV
Time between transitions
Constant time
Linearly Increasing time
Linearly Decreasing time



Energy vs Time
500000
50000
5000
500
50
5
eV
Nuclear Processes
Meta
Constant time
Linearly Increasing time
Linearly Decreasing time
Chemical Processes
Time between transitions

Klein-Gordon Equation

$$R'' + \frac{2}{\rho}R' + \left(\frac{\lambda}{\rho} - \frac{1}{4} - \frac{l(l+1) - \gamma^2}{\rho^2}\right)R = 0$$

$$xy'' + 2y' + \left(n^* - \frac{k-1}{2} - \frac{x}{4} - \frac{k^2-1}{4x}\right)y = 0$$

$$E = \frac{e^2 p}{2} \pm mc^2; p = 1, 2, 3....$$

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Conclusions

- Fractional quantum states are shown to be possible in atomic hydrogen
- Extensible spherical shells of dynamic electrons $\delta(r-r(p))$?
- The radii would vary from 386 fm to 53 pm (a_o) up to 7.2 nm where ionization and excited Rydberg states would continue the process.
- The fractional states produce a stepladder of quantized energy levels ranging from chemical < 10 eV, to nuclear > 0.51 MeV bridging the Meta-Energy gap.
- This provides a multi-step means of producing electron capture seen in LENR experiments in plasmas and liquids and solids starting from Rydberg energy chemical processes.

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